Computation of Characteristic Impedance for Multiple Microstrip Transmission Lines Using a Vector Finite Element Method

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Abstract—**The total characteristic impedance is computed from the field solution generated by a vector magnetic field finite element method for several microstrip geometries. By making use of the power orthogonality of the modes, characteristic impedances are computed. Additionally, the existence of negative modal characteristic impedances is verified for certain multiconductor striplines. Circuit parameters which are generated using this new method are verified by results from the spectral domain technique.**

I. Introduction

Recently, the finite element method (FEM) has become popular in the solution of Maxwell's Equations for microstrip [1]–[6]. However, the FEM generates field solutions which are not useful for the circuit designer. If a particular microstrip geometry is solved using a FEM code, the field quantities and dispersion information must be further processed to generate the characteristic impedance. This processing, as will be shown, involves computation of modal powers and strip currents, along with some basic matrix manipulations.

The FEM used here is based on a two dimensional potential energy formulation in terms of the magnetic field vector $\boldsymbol{H}$ [7]. Additionally, the energy functional contains a penalty function parameter which shifts low order spurious modes out of the propagation diagram [8]. Second order Lagrange interpolation polynomials are used over six node triangular elements. When a perfect electric conducting (pec) knife edge is encountered in the solution domain, singular edge functions are incorporated in the set of interpolating polynomials in order to hasten convergence [4], [5]. After the assembly of the triangle elements, a matrix eigenvalue problem is constructed and solved by using a sparse block iteration method for the first few eigenmodes [9]. The modal solution is in terms of the frequency of operation (k_o) and the three components of the magnetic field intensity at the triangle nodes. From the $\boldsymbol{H}$-field and the corresponding dispersion information, the three components of the electric field and the Poynting vector can be computed.

The advantage of using a finite element approach (based on a differential equation formulation) to produce circuit parameters lies in the fact that the solution technique is very general. A field solution is possible for virtually any waveguide regardless of the cross-sectional geometry, including transmission lines which support transverse electromagnetic (TEM), quasi-TEM and non-TEM waves. (Characteristic impedance can be uniquely defined for pure TEM waves only.) However, when the transmission line of interest supports a zero cutoff frequency quasi-TEM mode, as in the case of microstrip, a suitable characteristic impedance can still be defined. The problem with microstrip is that over the frequency range of interest, the definition of characteristics impedance can become ambiguous since the quasi-TEM mode becomes less like an ideal TEM mode as frequency grows large. The several possible definitions of characteristic impedance will each give different values.

If circuit parameters like characteristic impedance are to be produced, then it is necessary to find the modal strip currents and powers for all quasi-TEM characteristic solutions which a structure supports. By integrating the tangential magnetic field on the surface of the strips, longitudinal and transverse strip currents can be found. With reference to Fig. 1, by integrating the z-component (longitudinal component) of the Poynting vector over the waveguide cross-section, the total propagating power can be computed.

For this work, the TEM power-current orthogonality definition of characteristic impedance proposed by Weimer and Jansen [10] is employed for N conductor lines supporting $N - 1$ quasi-TEM modes. This is done because currents and powers are more easily found (computationally speaking) than voltages from the FEM solution.

The novelty of this work lies in the use of a general two dimensional finite element field analysis package (which models any waveguide geometry which is longitudinally invariant) to generate currents, voltages and powers and thereby compute normalized circuit quantities like characteristic impedance and velocity factors. Specialized

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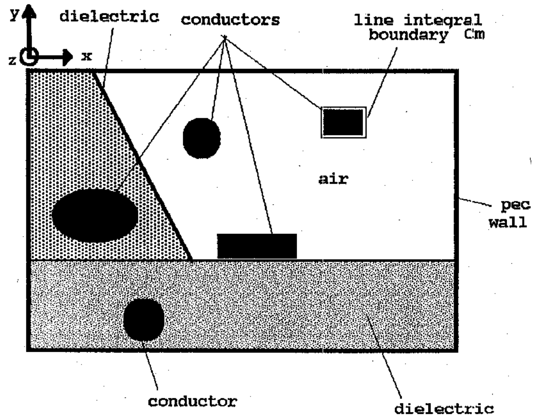

Fig. 1. Illustration of a general multiconductor transmission line with inhomogeneous dielectrics. A contour C_m is illustrated as an example line integral path for the evaluation Ampere's law.

methods of analysis like the spectral domain technique [11] require *a priori* information on the geometry of the problem to be programmed into the code, i.e., the Green's function for microstrip. On the other hand, the FEM requires no previous assumptions on the problem geometry other than the assumption that the problem domain must be finite in extent (usually enclosed by a perfect electric wall). Even though this paper focuses on the multiple microstrip problem, many other geometries can be characterized in terms of lumped circuit parameters using the approach developed here.

II. Formulation

Characteristic Impedance: There are two types of characteristic impedance which will be computed here. The first (and perhaps the most useful) definition of characteristic impedance is $\boldsymbol{Z_c}$, the total characteristic impedance. This quantity takes the form of an $(N - 1) \times (N - 1)$ matrix for an N conductor transmission line. In the case of shielded microstrip structure, there would be $N - 1$ strips and one shield. The matrix elements Z_{cij} relate the voltage on strip i to the current strip j. The second definition of characteristic impedance which is computed here is the modal impedance, $\boldsymbol{Z_m}$. The elements of this matrix relate the voltage to the current on a given line for each of the quasi-TEM modes that exist.

It is known that an N conductor structure will support $N - 1$ orthogonal quasi-TEM modes (i.e., the field eigenfunctions are orthogonal). Following Marx [12], Weimer and Jansen [10], it can be inferred that given the complete sets of current eigenvectors $\boldsymbol{I}$ and voltage eigenvectors $\boldsymbol{V}$ derived from the orthogonal field wavefunctions will satisfy,

$$\boldsymbol{I}^T\boldsymbol{V} = \boldsymbol{P} \tag{1}$$

where $\boldsymbol{P}$ is diagonal and element P_k^k is the propagating power for mode k and $[\cdot]^T$ represents the transpose of a matrix. Expression (1) is the heart of the characteristic impedance computation. Using Ohm's Law, it can be shown that

$$\boldsymbol{I}^T\boldsymbol{Z_c}\boldsymbol{I} = \boldsymbol{P} \tag{2}$$

where $\boldsymbol{Z_c}$ is the total characteristic impedance matrix. By performing some matrix inversions and multiplications, $\boldsymbol{Z_c}$ can be explicitly calculated from the modal powers and the eigencurrents. It is interesting to note that the characteristics impedance matrix is nothing but the diagonal modal power matrix that has undergone a change of basis via the symmetric similarity transformation

$$\boldsymbol{Z_c} = [\boldsymbol{I}^{-1}]^T\boldsymbol{P}\boldsymbol{I}^{-1}. \tag{3}$$

Since $\boldsymbol{P}$ is real, positive and diagonal and $\boldsymbol{I}$ is real, the total characteristic impedance matrix must be symmetric and real. The symmetry of $\boldsymbol{Z_c}$ is indicative of the reciprocal nature of the transmission lines examined here and the fact that $\boldsymbol{Z_c}$ is real is a consequence of the lossless approach to the analysis taken here.

After computing the total characteristic impedance, the modal characteristic impedance matrix, $\boldsymbol{Z_m}$, can be easily produced. The modal characteristic impedance, as defined in [11]–[13] is

$$Z_{mik} = \frac{V_{ik}}{I_{ik}}. \tag{4}$$

The matrix element Z_{mik} in (4) corresponds to the ratio of the voltage and the current on line i for mode k. This definition of characteristic impedance could be useful for studying the nature of mode propagation on multiconductor TEM and quasi-TEM transmission lines.

It is worthwhile to illustrate why some of the other methods for finding $\boldsymbol{Z_c}$ are not used. Two methods which have enjoyed widespread use for many years are the voltage-current formulation and the partial power method [13]. Both of these methods are just as valid as the power-current method for TEM lines. However, in the case of inhomogeneous dielectrics which no longer support TEM modes, there is no unique value that can be found for voltage and current. However, if a convenient integration path could be found where the fields look almost like TEM fields, reasonable approximations could be made for the current and voltage. For evaluating the current, integration contours are taken around the surface of the conductors. At lower frequencies, the quasi-TEM modes are primarily TM_z (of which the TEM_z modes are a subset). Near the conductors, therefore, the current integral around the conductors should yield fairly unique values for current. As frequency increases, however, the quasi-TEM modes will become more TE_z and the current definition becomes more ambiguous. That is to say, the longitudinal magnetic field intensity (or transverse current density) becomes significant and Ampere's law does not provide a unique value for the current.

Perhaps the largest inconvenience in the V-I method is the necessity of computing a voltage integral. Unlike the current integral, no convenient path of integration exists for the voltage. Only in zero-frequency cases does the voltage integral produce a valve value independent of path. The voltage integral is more path sensitive than the current integral for nonzero frequency (as is evident when one considers the difference in electric field strength in a microstrip substrate and out of the substrate). For this reason, the V-I method of finding characteristic impedance is not used.

The partial power technique for the determination of modal impedance is not used here for a couple of reasons. One reason is that the computation of the required integral is inconvenient due to the nature of the finite element field solution technique. The partial power technique requires the evaluation of

$$Z_{mik} = \frac{\iint E_k \times H_{ik} \cdot dS}{I_{zik}^2}, \quad (5)$$

where E_k is the modal electric field profile and H_{ik} is the magnetic field for mode k due to the current on line i. Z_{mik} in (5) represents one definition of the modal impedance based on (4). The inconvenience is that it is much too difficult to solve for the individual contributions of the line currents to the magnetic field using the FEM. The partial power technique is most often used with the spectral domain (integral equation) technique, where partial modal powers are easily obtained by inserting the line currents one by one into Green's function equation and computing the magnetic field for the single line. In the spectral domain technique, the fields are found from the currents. On the other hand, in the FEM the currents are found from the fields. This makes the computation of any type of "partial field" from a single strip current difficult. Consequently, a unified method of computing all characteristic impedance matrix elements directly from the field-derived quantities (mode powers and current eigenvectors) is needed. Additionally, the partial power method does not implicitly enforce the orthogonality of the eigencurrents and eigenvoltages which exists for TEM modes, and is enforced for the quasi-TEM modes in (1).

Power and Current Computation from the Fields: The numerical implementation of the characteristic impedance algorithm involves converting the FEM field solution into powers and currents. The goal is to compute the currents on all $N - 1$ microstrips for all of the $N - 1$ quasi-TEM modes. Using Ampere's Law, the longitudinal conductor currents in the general multiconductor transmission line structure of Fig. 1 are found by

$$\oint_{C_i} H_k \cdot dl = I_{zik}, \quad (6)$$

where the contour C_i is taken to be a path just outside (by some infinitesimal distance) the ith current carrying conductor. In (6), H_k is the total magnetic field intensity for mode k and I_{zik} is the longitudinal current on strip i for mode k. This integral is evaluated using the same Lagrangian and singular edge interpolation functions which are used to construct the functional equation in the FEM. The integrals of the FEM shape functions can be evaluated analytically in terms of the contour lengths and the nodal values for the transverse magnetic field. For the second order Lagrange polynomials, an exact analytic solution for the integrals reduce to Simpson's Rule. The integrals of the edge functions are also analytically expressed. Since the transverse field near an edge varies as $r(\pi/\alpha) - 1$, where r is the distance from the edge and α is the span angle of the edge [14], it can be seen that this integral in terms of r is easily evaluated. The edge functions are necessary only inside of the elements which share a node on a metal knife edge. All that is needed is the length of the line segment over which the integral is defined and the coefficient of the edge function (which is provided by the FEM eigensystem solution).

The interior region microstrip conductors are assumed to be polygonal perfect electric conducting (pec) boundaries with two or more convex edge nodes. The edge nodes correspond to the vertices of the polygon. The nodal transverse magnetic field values are inserted into the analytical expression for the line integral along the line segment which defines each side of the polygon. This type of formulation allows faster computation of the eigencurrent matrix for multiple conductor problems than a brute force numerical integration of the field eigenfunction along the outside of the strips.

The propagating power for all the modes of interest following from the integration of the Poynting vector, which from modal orthogonality gives

$$\iint_{\sigma} E_j^* \times H_k \cdot dS = P_k \, \delta_{jk}. \quad (7)$$

In (7), σ is the cross-section of the entire structure, j and k are the mode numbers, E_j and H_k are the modal electric and magnetic field vectors for the jth and kth modes, respectively, P_k is the longitudinally propagating power for the kth mode, and the δ_{jk} is the Kronecker delta function.

The same shape functions are used in the power computation as in the construction of the FEM global matrix equations. This proved to be the best method for evaluating the power integrals, since the universal matrix technique described by Silvester [15] could be used for the Lagrange function integration. The edge functions are integrated by using the Gauss–Legendre method. The need for two types of integration increased the complexity of the code but was necessary to maintain accuracy.

Since the FEM formulation is based on a solution for the magnetic field, (7) must be put into a form based on the magnetic field vector. Using Maxwell's equations, the modal power can be written as

$$P_{ek} = -\frac{1}{j\omega\epsilon_0} \iint_{\sigma_e} H_k \times [\epsilon_r]^{-1} \nabla \times H_k^* \cdot dS. \quad (8)$$

where P_{ek} is the contribution to the propagating power from element e for mode k, σ_e represents the surface of element e, and dS is the differential surface element oriented with its normal in the longitudinal (propagation) direction. Given (8), the total propagating power can now be expressed as the sum of all the elemental contributions, so

$$P_k = \sum_{e=1}^{E} P_{ek} \tag{9}$$

represents the total propagating power in the transmission line for E mesh elements in the discretized solution domain.

In terms of the software implementation of (8) and (9), the most difficult aspect is the actual computation of (8). There are two types of basis functions used in the FEM code, Lagrange polynomials and edge functions. Since the elements which do not lie on a sharp convex pec edge use Lagrange functions and the elements which lie on edges use both Lagrange functions and edge functions, a formidable bookkeeping job results.

In general, the magnetic field is determined by

$$\boldsymbol{H}_{ek} = \sum_{i=1}^{6} [\boldsymbol{H}_i]_{ek} N_i(x, y) + C_{ek}\boldsymbol{g}(x, y), \tag{10}$$

where $[\boldsymbol{H}_i]_{ek}$ is the nodal magnetic field intensities for element e and mode k, C_{ek} is the edge coefficient for element e and mode k (if there is no edge element e, then C_{ek} is zero), and $N_i(x, y)$ and $\boldsymbol{g}(x, y)$ are the Lagrange and edge basis functions, respectively. In the case where there is no edge function present in (10), (8) is evaluated by using the previously mentioned universal matrix technique. If C_{ek} is nonzero, then a numerical solution of (8) by the Gauss–Legendre method is necessary to find the contribution from the edge function. This adds extra overhead to the computation time, but the gain in accuracy by including edge function contributions overshadows the increase in program complexity.

III. Example Calculations

In order to validate the preceding methodology, several example calculations using three microstrip geometries are presented. The simplest example is a single microstrip line which is 1.0 mm wide on a 1.0 mm thick substrate with $\epsilon_r = 10.0$, as shown in Fig. 2. This configuration is known to have a characteristic impedance of about 50 Ω at low frequencies. The velocity factor β/k_0 is found to be 2.59 using a TEM approximation given by Bhartia and Bahl [16]. It is expected that the transmission line circuit parameters (β/k_0 and Z_c) will be slowly varying functions with respect to frequency. Note that according to the definitions of total characteristic impedance and modal characteristic impedance, $Z_c = Z_m$, since there is only one strip and, therefore, only one quasi-TEM mode. Figs. 3 and 4 show the velocity factor and characteristic impedance curves, respectively, for the single strip geometry. Domain method produced results, shown on these curves, verify the trend of the solutions produced by the FEM even though the points do not match exactly. The differences between the two methods can be explained by considering the different convergence properties of the two

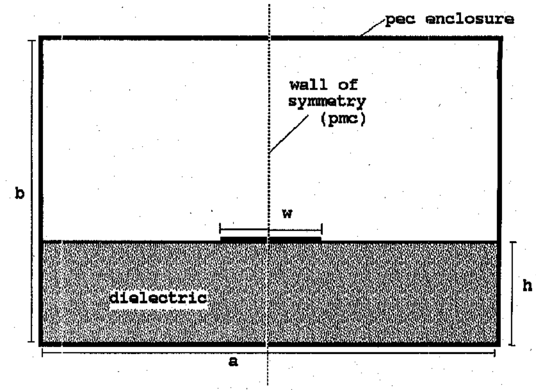

Fig. 2. Single strip geometry discussed in the text with a = 10.0 mm, b = 5.0 mm, h = 1.0 mm, w = 1.0 mm and ϵ_r = 10.0.

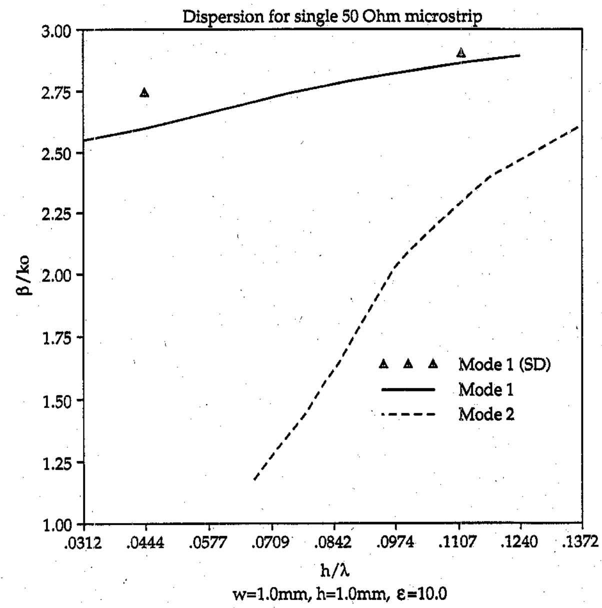

Fig. 3. Dispersion relationship for the first two modes in the single microstrip problem. The solid line corresponds to the microstrip mode. The triangles represent the spectral domain solution.

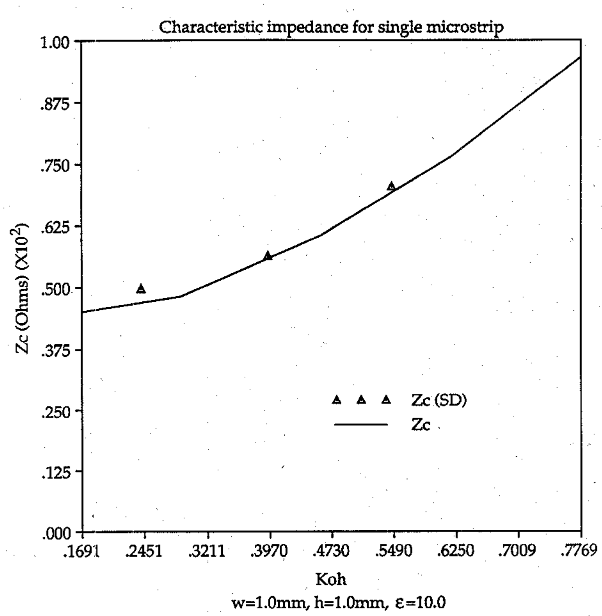

Fig. 4. Characteristic impedance of the single strip is presented here. Triangles represent the spectral domain solution.

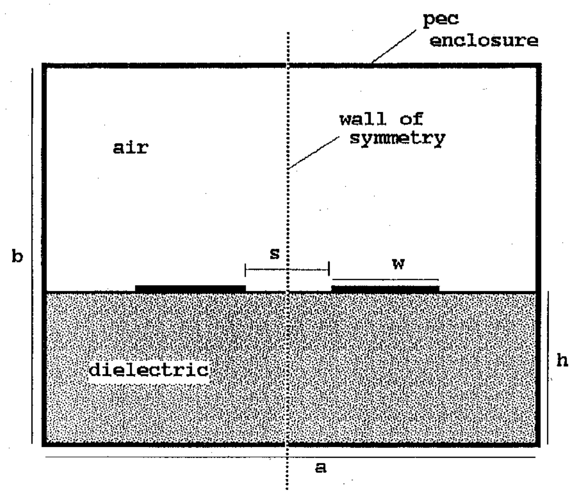

Fig. 5. Dual symmetric single plane microstrip geometry with a = 10.0 mm, b = 5.0 mm, h = 1.0 mm, w = 2.0 mm, s = 1.0 mm and $\epsilon = 4.0$.

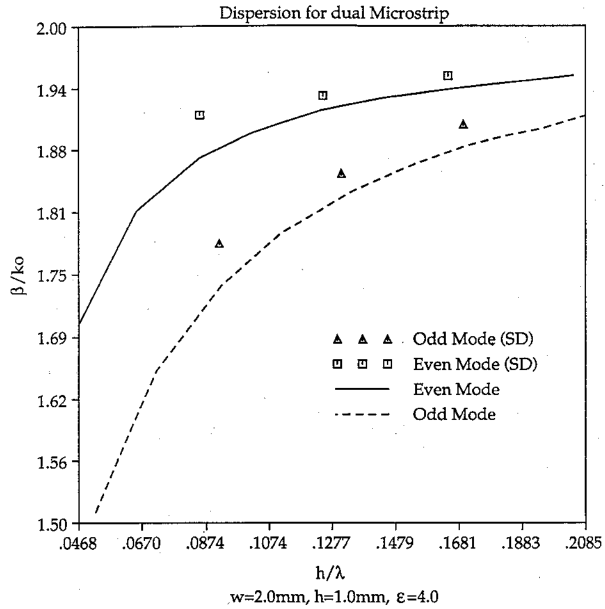

Fig. 6. Dispersion curves for the even and odd modes of the dual symmetric single plane microstrip.

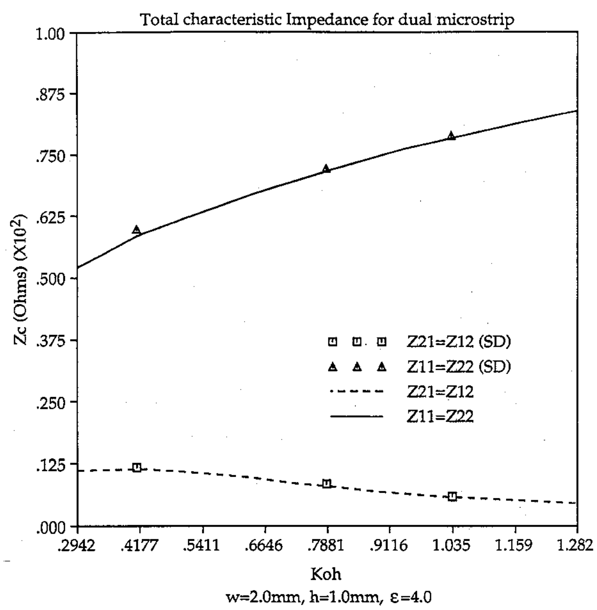

Fig. 7. Dual strip total characteristic impedance curves.

methods. For example, the FEM will begin to exhibit errors in the eigensystem solution as the frequency goes to zero due to the increasing condition number of the global matrices and the limitations of the interpolation. For the single strip example, the solution domain is discretized into 720 triangles, which corresponds to about 4500 unknowns in the matrix eigenvalue problem. The mesh is almost uniform over the waveguide cross-section, with the smallest elements being under the microstrip. The ratio of areas between the largest and smallest element is not greater than 4. The solution of this problem takes about 25 min on an Ardent Titan P-3 super-minicomputer, with about eight megabytes of core storage being required. As in all the examples which are presented here, the bulk of the computing time is utilized by the eigensystem solver. The matrix assembly and I/O operations are very fast by comparison.

The eigensystem solver based on the subspace iteration method [9] is worthy of note here. This method allows for the computation of a small number of the extreme eigenstates of a very large (orders greater than 10 000) matrix eigenvalue problem. Since the FEM formulation here is based on a differential equation based energy operator, the matrices are sparse and the average bandwidth is small. Sparsity occurs because the value of the functional at any point depends only on the field intensities at the nearest node points. For the problems solved in this paper, about one percent of the matrix elements in the global finite element matrices needed to be stored, or about 10^6 double precision numbers in the single microstrip problem.

Fig. 5 shows a dual strip structure which is symmetric about the center of the shield. The dimensions are: strip width = 2.0 mm, substrate height = 1.0 mm, strip separation = 1.0 mm. The substrate dielectric constant is $\epsilon_r = 4.0$. These striplines are found to have self impedances of about 50 Ω (for large strip separation) and a velocity factor of 1.76, using a TEM approximation [16]. The dispersion curves for the mode velocity factors, the total characteristic impedance matrix $\boldsymbol{Z}_c$, and the modal characteristic impedance matrix $\boldsymbol{Z}_m$ are presented in Figs. 6–8, respectively. Note that all components of modal characteristic impedance are positive for this example. Again, the spectral domain method is used to verify the results in Figs. 6–8. It has been shown by Carin [11] that all modal characteristic impedance matrix elements should be positive when symmetric single plane dual striplines are considered. The discretization in this example used the same number of elements over the solution domain as the single strip example. Again, 720 triangular elements are used in a fairly uniform mesh for the dual strip.

The last structure presented is a dual layer, dual microstrip geometry designed to illustrate the existence of

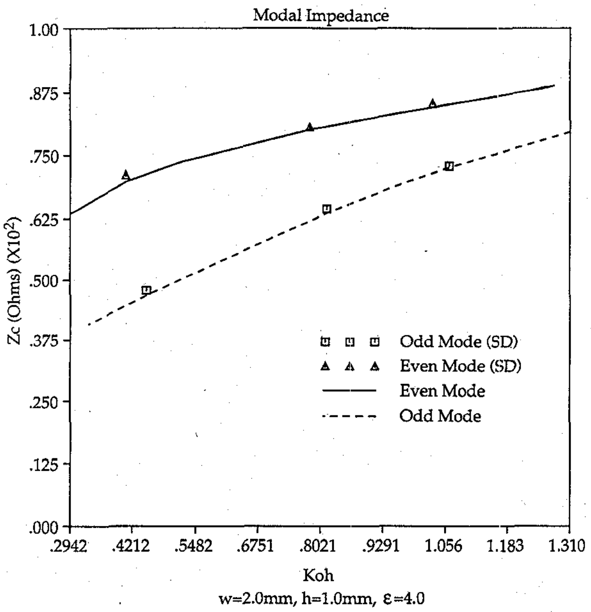

Fig. 8. Dual strip modal characteristic impedance curves.

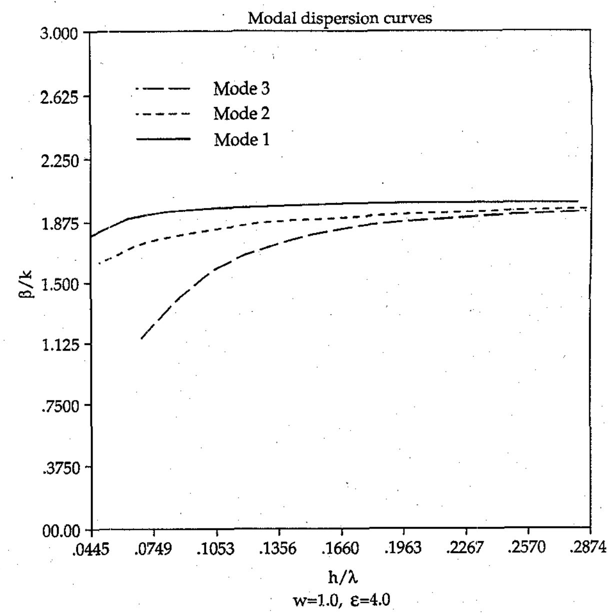

Fig. 10. Dispersion curves for the two microstrip modes of the structure illustrated in Fig. 9.

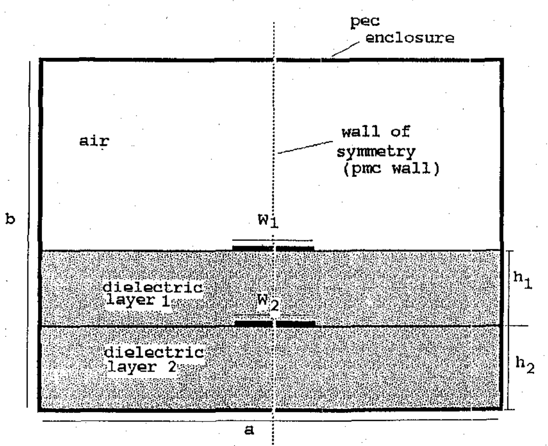

Fig. 9. Dual strip dual plane microstrip geometry is shown with $a = 10.0$ mm, $b = 8.0$ mm, $h_1 = h_2 = 1.0$ mm, $w_1 = w_2 = 1.0$ mm and $\epsilon_r = 4.0$ for both dielectric layers.

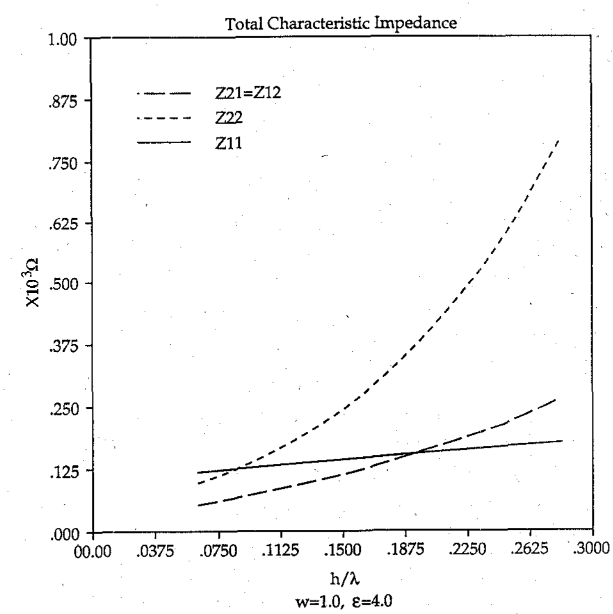

Fig. 11. Total characteristic impedance curves for the dual plane structure illustrated in Fig. 9.

negative modal characteristic impedances. It is shown in [11] that for a two strip geometry if

$$\frac{P_1}{P_2} I_{z12} I_{z22} < -I_{z11} I_{z21}, \tag{11}$$

then negative off-diagonal elements of the modal characteristic impedance will occur [11]. It is worth reiterating that the total characteristic impedance will always be positive, real and symmetric for lossless, reciprocal passive transmission lines (in order to satisfy the conservation of energy law). The microstrip transmission line dimensions are illustrated in Fig. 9 and are given as: strip widths = 1.0 mm, first layer thickness = 1.0 mm, and second layer thickness = 1.0 mm. As in the previous examples, the substrate dielectric constant is taken to be $\epsilon_r = 4.0$. The plots displayed in Figs. 10–12 show velocity factor, total characteristic impedance and modal characteristic impedance for the dual plane geometry. The values for Z_{m11}, Z_{m22} and Z_{m12} agree well with those generated by the spectral domain technique. (Spectral domain calculations performed for an operating frequency of about 9 GHz.) The FEM is seen in Fig. 12 to yield a value of about $-200\ \Omega$ for Z_{m21} while the spectral domain method gives a value of $-750\ \Omega$. This is because the current on line two is very small for mode 1. Small perturbations in the current solution on line two will generate large errors in the modal impedance.

The geometry is discretized into 1232 triangular elements, which presents a matrix eigenvalue equation of order 7500 to the eigensystem solver. It takes about 50 min of CPU time on the Stardent Model P3 computer to solve for the fields in this structure. The required core storage comes to about 18 Mbytes and as in the previous exam-

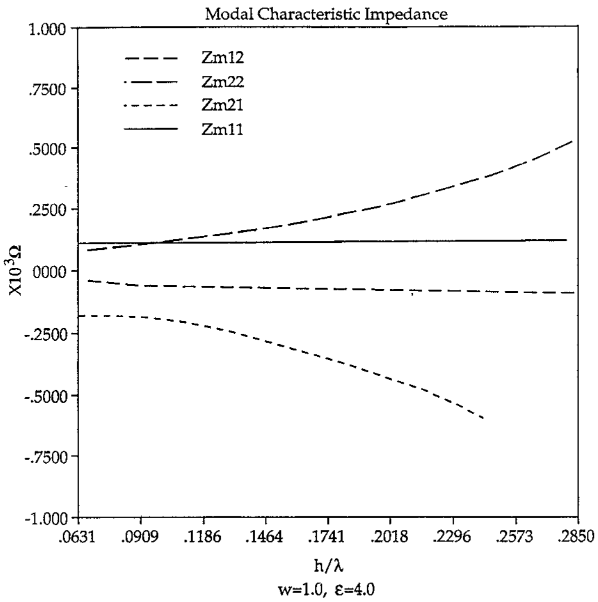

Fig. 12. Modal characteristic impedance curves for the dual plane structure in Fig. 9. See text for discussion on the comparison with spectral domain data.

ples most of the CPU time is devoted to solving the matrix eigenvalue problem.

IV. Conclusion

A straight forward method has been found to solve for interconnect circuit parameters using the results of a vector differential equation formulation with a finite element solution. This work is unique in the sense that an FEM field solution is used to compute circuit quantities for quasi-TEM transmission lines. In general, this method can solve problems with any number of conduction wires inside of an enclosing perfect electric boundary of any shape. Inhomogeneous and anisotropic dielectric materials can also be conveniently handled using the FEM. The modal and total characteristic impedance of a transmission line are found by using elementary matrix transformations on the current eigenvectors and the modal powers. The current eigenvectors are computed from the magnetic field eigenstate through the use of Ampere's law and the modal powers are found by integrating the Poynting vector over the cross-section of the shielded domain. The linking of the FEM with a circuit CAD program such as SPICE is now possible.

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